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Enhancing Supply Chain Coordination Using Mathematical Modeling

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Abstract

Effective Coordination across different entities of a supply chain is a critical factor in improving overall efficiency, reducing costs, and increasing responsiveness to market dynamics. This study presents a mathematical model to enhance supply chain coordination by optimizing order and resource allocation among suppliers, manufacturers, and distributors. The proposed model integrates key decision variables, production quantities, transportation capacities, and inventory levels into a mixed-integer Linear Programming (MILP) formulation. By minimizing total system costs and balancing the objectives of all partners, the model promotes cooperative decision-making and reduces the bullwhip effect. The model's performance was validated using numerical experiments and sensitivity analyses, demonstrating its ability to improve coordination efficiency and stability under varying demand and supply conditions. The data were evaluated using expert opinion and a mathematical equation in GAMS. The results showed that the Coordination of contractor and supplier orders is mainly in the sensitivity analysis group, as reflected in the objective function values. Moreover, the proposed model showed that optimizing order management and unit earnings based on it has functional and developmental capabilities.

Keywords: Supply chain, Mathematical model, Coordination.

1 | Introduction

In today's highly competitive and globally interconnected markets, supply chain coordination has become a critical determinant of organizational success. The performance of a supply chain depends not only on the efficiency of individual entities—such as suppliers, manufacturers, distributors, and retailers—but also on the degree of coordination and information sharing among them [1]. Lack of coordination often leads to inefficiencies, including the bullwhip effect, excess inventory, high operational costs, and poor responsiveness

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to market fluctuations. Therefore, achieving effective coordination mechanisms is a key challenge and a strategic priority for modern supply chain management [2]. Nowadays, an organization's supply chain performance relative to rivals is a vital competitive tool across industries [3]. Given globalization and the rapidly changing business landscape, the supply chain has become a tool for organizations seeking to compete effectively, both locally and globally. Overall, decision-making in the supply chain is done both centrally and decentralized. In a centralized supply chain, a single decision-maker, or the main member, who has access to sufficient supply information and the power to make decisions, serves as the policy-maker for the whole chain [4]. An organization's competitive advantage largely depends on its supply chain. One of the key concepts of the past few decades is supply chain management. The most important reason for this focus is the ever-increasing competitiveness and efforts to survive within organizations. In recent decades, this has led to the management of procurement, production, and distribution processes to take steps towards the organization's competitive objective. In a supply chain, raw materials are sourced from suppliers and delivered to retailers or end users. The supply chain comprises all sectors directly or indirectly involved in fulfilling customer orders. Moreover, as markets become more competitive, managing customer orders becomes increasingly important [5]. One of the crucial functions that supply chain manufacturing companies are always concerned with, and the decisions related to it significantly contribute to their competitiveness, is supply chain management [6]. Today, intense global competition has led to increased customization and faster customer response. Growing attention to supply chain performance shows the importance of relying on specific supply chain management practices. Supply chain coordination is a mechanism for aligning supply chain members around general decisions to achieve optimal profitability for the entire supply chain [7].

Mathematical modeling has emerged as a powerful tool to address complex coordination problems in supply chains. By representing interrelated decisions and constraints through analytical formulations, mathematical models enable decision-makers to optimize resource utilization, synchronize operations, and evaluate trade-offs among conflicting objectives [8]. Approaches such as Linear Programming (LP), Mixed-Integer Linear Programming (MILP), and nonlinear optimization have been extensively used to design coordination policies, improve order management, and align incentives across the supply chain [9].

Despite significant advancements, many supply chains still operate under suboptimal coordination structures due to demand uncertainty, information asymmetry, and decentralized decision-making. This study aims to develop a mathematical model that enhances coordination among supply chain partners through optimized order allocation and information flow [10]. The proposed model focuses on minimizing total system costs while meeting service-level requirements and balancing the interests of all stakeholders. By integrating quantitative optimization techniques with coordination mechanisms, the research provides both theoretical insights and practical implications for improving overall supply chain performance [11].

This study uses LP to develop a mathematical model for optimizing and coordinating the order management problem in the supply chain. To more precisely specify the problem parameters for decision-making, we proposed a supply chain structure for similar industries, covering operations, materials, supplies, and orders. The rest of the paper is organized as follows. In the second part, the literature review, and in the third part, the problem will be presented. In the fourth part of modeling, in part five, the results based on the decision variables; in the sixth section, the results based on the sensitivity analysis (the aspect of innovation and coordination of the order in the study); and in section seven, the discussion and conclusions are reported and examined.

2 | Literature review

Chaharsoughi and Heidari [12] examined a decentralized supply chain with a buyer and a supplier, in a multi-period context, where demand and supply are uncertain. The proposed model shows that simultaneous coordination of the reorder point and the order quantity can reduce supply chain costs and fairly allocate profits based on each member's bargaining power. Chaharsoughi et al. [13] presented a coordination model for discounts on the coordination of the order quantity and reorder point in a two-level supply chain. They understood that coordinating the reorder point and order quantity could improve supply chain profitability and customer service. Arkan and Hejazi [14] presented a coordination mechanism based on the credit payment period in a two-level supply chain with one buyer and a supplier, designed to provide the buyer with a normal distribution of potential delivery times, a scheduled delivery time, and ordering costs. Heidari (2014) presented a two-level supply chain in three decentralized, non-coordinated, centralized and coordinated modeling ways according to a new mechanism based on "controlling undetermined issues of delivery times" and attention is paid to encourage retailers to coordinate the reordering point with respect to the profitability of the entire supply chain, which can be effective in changing its decisions regarding optimal head-to-head supply chain decisions. Asadia and Abolghasemian [2] used stochastic programming to incorporate demand uncertainty and risk management in coordination decisions. Rezaei et al. [15] combined mathematical programming with metaheuristics to solve large-scale, multi-stage coordination problems. Ghasemi and Abolghasemian [8] introduced a mathematical model for coordination under flexible manufacturing and stochastic demand conditions. Iraj et al. [1] investigated collaborative contracts to improve profit sharing and coordination efficiency among supply chain members. Liotta et al. [16] proposed a simulation–optimization approach for integrated coordination of production, transportation, and sales. Pérez-Perales et al. [17] presented a mathematical programming-based framework for evaluating collaborative planning scenarios.

Although substantial progress has been made in modeling supply chain coordination, several research gaps remain. Many existing models are limited to specific stages of the supply chain or assume centralized control, which may not reflect practical conditions. Moreover, the integration of multiple coordination mechanisms—such as information sharing, joint order allocation, and cost optimization—within a unified mathematical framework is still underdeveloped.

This research contributes to the literature by proposing a comprehensive mathematical model that enhances coordination among supply chain partners through optimized order management and cost-sharing strategies. The model aims to minimize total system costs while ensuring balanced decision-making among decentralized actors. By bridging theoretical modeling and practical implementation, the study advances the understanding of how mathematical optimization can be effectively applied to real-world supply chain coordination.

3 | Materials and Methods

Imagine a supply chain. The flow of goods, sources, and services constantly exchanges between the members of the chain. If the supply chain in question has a contractor and a supplier, each should have a profit or benefit at the end of each exchange. Thus, the issues or factors that can be exchanged and are decisive are numerous. Earnings or profits earned may be separate or single. According to preliminary studies, many factors, such as raw materials, parts, supplies, and orders, can shape the relationship between the contractor and the supplier and generate profits. The research criterion in this study was to determine the supply chain's unit profit. For having this axis, the topic is required by the experts, the basis of this proposed model of supply chain orders, as well as other factors mentioned in the forthcoming model. Thus, to analyze optimality and achieve single-unit profit, we will coordinate supply chain orders. The problem overview is shown in *Fig. 1*.

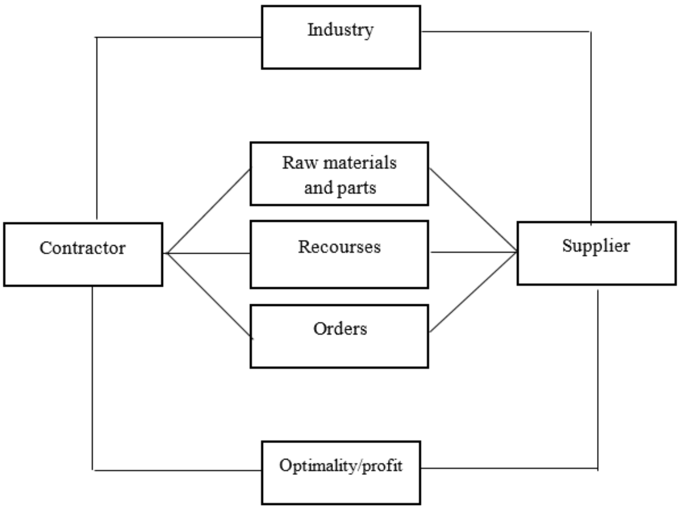

Fig. 1. The overview of the problem and the researcher-made conceptual model (2017).

To examine the research bases of this study and to collect data, 50 experts (active in various industries) will be interviewed. As the proposed model in this paper is based on LP, it is necessary to introduce the indices, parameters, and decision variables to formulate the factors. *Tables 1-3* show these, respectively.

Table 1. Indices reviewed by experts.

Index	Features
S	Contractors
L	Suppliers
K	Raw materials and parts
R	References
I	Orders

Table 2. The parameters examined by the experts.

Parameters	Features
P	The proposed price S for the workload of order i, within source r
p´	The proposed price of supplier L for part or raw materials K
SE	The value of source r allocated to order i during the period t assigned by the contractor s
SO	The amount of raw materials and parts of the order i supplied by the supplier L during the period t
IW	The input workload corresponding to the order i on the source r at time t
CR	Maximum available source r at normal time in period t
CS	The maximum capacity for outsourcing the source r during period t by the contractor s
PR	Maximum capacity of supplier I in period t
q	Probability of acceptance of i order by a customer in period t
CO	The expected defective rate for supplier L during period t

Table 3. Decision variables studied by experts.

Decision Variables	Features
y	The value of source r allocated to order i in the period t
X	If contractor s meets the required workload of order i on the source r, otherwise it is zero
X'	If the supplier L supplies the raw material and part k required by order i, otherwise it is zero

After showing the research items used, we will use LP to coordinate supply chain orders.

3.1 | Objective Function

The objective function of the problem, presented as a LP problem, is to minimize the average of total production costs, operating costs, parts purchase costs, raw materials and labor costs, and sourcing costs, while accounting for required order quantity, period, and inside outsourcing.

3.2 | Constraints

The values of s=1, L=1, K=1,2, R=1,2, I = 1,2, and T=1,2, were determined according to expert opinions. These values show that the research problem involves a contractor, a supplier, two types of material or parts, two sources, two order periods, and one period. Somehow, the hypothesis is based on the fact that periodic studies are related to the financial years 2015 and 2016 in similar state sources in terms of materials, sources, order, and time period, and may change over the years. The parameters and their values are summarized in the tables below. Prices are in Toman. Capacities are calculated in tons.

Table 4. The value of parameter P.

Parameter	Contractor 1		
	Order 1		Order 2
P	Source 1	2000	3000
	Source 2	2000	3000

Table 5. The value of the parameter p´.

Parameter	Supplier 1	
p´	Raw material 1	1200
	Raw material 2	1500

Table 6. The value of the parameter SE.

Parameter	Contractor 1			
	Period 1		Period 2	
SE	Order 1	100	100	Source 1
		100	100	Source 2
	Order 2	150	150	Source 1
		200	200	Source 2

Table 7. The value of parameter SO.

Parameter	Supplier 1		
	First period		Second period
SO	Order 1	250	250
	Order 2	300	300

Table 8. The value of parameter IW.

Parameter	First Period		Second Period	
IW	Source 1	270	270	Source 1
	Source 2	270	270	Source 2

Table 9. The value of parameter CR.

Parameter	First Period		Second Period
CR	Source 1	4500	4500
	Source 2	4500	4500

Table 10. The value of parameter CS.

Parameter	Contractor		
CS	First period		Second period
	Source 1	4000	4000
	Source 2	4000	4000

Table 11. The value of parameter PR.

Parameter	First Period		Second Period
PR	Order 1	0.9	0.9
	Order 2	0.9	0.9

Table 12. The value of parameter q.

Parameter	Supplier	
q	First period	0.01
	Second period	0.01

Table 13. The value of the parameter CO.

Parameter	Supplier	
CO	First period	0.01
	Second period	0.01

The analysis was implemented in the Gams software.

4 | Results Based on Decision Variables

After entering the information into GAMS, we could analyze the behavior and states of the decision-making variables for each index and identify the optimal point. *Table 14* contains variables y, x, and x` showing i for 1 and 2, t for 1 and 2, k for 1 and 2, s for 1, and L for 1.

Table 14. The results based on the decision variable.

$y_{111} = 972$	$y_{121} = 972$
$X_{111} = 1$	$X_{221} = 1$
$X'_{121} = 1$	$X'_{221} = 1$
Z=8744	

The description of each of the states is as follows:

$y_{111} = 972$: this means that for the first order and source, the value of y is 972 tons in the first period.

$y_{121} = 972$: this means that for the first order, for the second source used, and for the first period, the value of y is 972 tons and equal to the first state.

$X_{111} = 1$: this means that, for the first order, the first source used, and the single contractor, the problem has 1 workload.

$X_{221} = 1$: this means that, for the second source used and the single contractor, the problem has 1 workload.

$X'_{221} = 1$: this means that the first order is used for the second part, and for the single supplier of the problem, the value of x` is 1 of the supply.

$X'_{121} = 1$: this means that the second order for the second piece used for the single supplier of the problem, the value of x` is 1 of the supply.

It is noted that the other states of the problem have not been analyzed due to the absence of responses. It also shows the optimal value of this problem.

5 | Results Based on Sensitivity Analysis on Results

Sensitivity analysis shows the effect of changes in a model's inputs on its outputs. Table 15 presents the sensitivity analysis of the study's results. Since the problem is coordination between contractor and supplier orders, the decision was made to analyze the sensitivity of the two supply chain members. In doing so, for the validation of two parameters arbitrarily chosen from -50 to 300+, the effects of these numbers are measured in the function (the selected range of numbers was determined in consultation with experts and without a specific rationale for -50, 0, 100+, 200+, 300+). The result was that if -50 units are subtracted from S0 and SE, then the objective function 8994 (zero interference) yields S0 and SE unchanged, the same as 8744. If +100 units plus S0 and SE are added to the objective function number 7444, if +200 units are added to S0 and SE, then the number of the objective function is 7044, and if +300 units are added to S0 and SE, then the objective function number will be 6650. This means that as we increase validation, the objective function value decreases, and as we lower the validation effect, the objective function value increases. So we will have:

The following table summarizes: coordination of orders is mainly dependent on sensitivity analysis.

Table 15. Sensitivity analysis on results.

Validation Parameters	Objective Function Value
SO, SE - 50	Z=8994
SO, SE	Z=8744
SO, SE+ 100	Z=7444
SO, SE + 200	Z=7044
SO, SE + 300	Z=6650

5 | Conclusion

Companies increasingly compete with each other over product variety and service aspects, including commitment to orders and delivery times. Quality and costs are essential criteria for market competition, so that precision in order commitment, delivery speed, and delivery frequency can make the order more successful. With parallel advances in technology, manufacturing systems adopt a new philosophy, enabling them to establish a solid relationship between customers and suppliers and, eventually, form an integrated supply chain. Moreover, the product life cycle is steadily decreasing, requiring more agile order management and greater flexibility. Furthermore, traditional code-order management models do not have a specific commitment to existing or future merchandise. Therefore, the key to timing such production systems is to provide a timely commitment to the customer. However, commitment to order and comprehensive scheduling criteria are usually not considered simultaneously, and companies' costing standards have so far not adequately addressed this issue, particularly when it can be profitable.

The operational capacity models used for order management activities are based on understanding process constraints, balancing the flow of materials through the production chain, and meeting the customer's needs. On the contrary, to produce for real-time orders, we need a profitability model and a commitment to effective orders to transform order acceptance and production scheduling into a joint decision-making process, thereby generating productive activities that create value and preventing or minimizing non-addictive activities. The supply chain begins with the customer's order and ends when the customer completes purchasing the goods and services. The difference between the money the customer pays and the total costs incurred by the chain to produce and distribute the goods is the chain's profitability. Supply chain management involves managing the flow between and within each step of the chain to record its overall profitability. Firstly, supply chain management considers any cost-cutting approach that helps meet customer needs (from suppliers and facilities to raw materials, distribution centers, retail stores, and warehouses). Secondly, supply chain management is increasing efficiency and reducing costs throughout the system. By using supply chain management approaches, the total system cost —including transport, inventory, transfer, and other costs — is reduced. However, it does not emphasize that transport and inventory costs will decrease. At the same time, supply chain management will seek to improve overall chain performance and increase customer service through a systematic approach. Supply chain management brings together the efforts of companies, suppliers, manufacturers, traders, and customers to meet customer needs. This approach first seeks to reduce the company's costs by optimally planning the production and supply of components and materials, ensuring the timely delivery of the required materials. Finally, production and inventory planning will increase the company's profitability. Supply chain management leads to closer relationships among value chain components, enabling higher-quality products to be delivered to customers in relevant markets, promptly, and at reasonable prices. The purpose of this approach is to develop strong relationships within the organization's supply chain so that they can act in an integrated way, fully integrate with business processes, and maximize customer satisfaction. If the supply chain offers no significant value to customers, and managers conclude that the organization alone cannot deliver significant improvements, they may consider employing supply chain management to deliver products and services faster, with better quality, and at lower cost.

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Data Availability

The data supporting the findings of this study are available from the corresponding author upon reasonable request.

References

[1] Iraj, M., Chobar, A. P., Peivandizadeh, A., & Abolghasemian, M. (2024). Presenting a two-echelon multi-objective supply chain model considering the expiration date of products and solving it by applying MODM. *Sustainable manufacturing and service economics*, *3*, 100022. https://doi.org/10.1016/j.smse.2024.100022

[2] Asadia, F., & Abolghasemianb, M. (2018). Review coordination of advertising policy and its effect on competition between retailer and manufacture in the supply chain. *Computational research progress in applied science and engineering*, *4*(3), 62–66. https://www.researchgate.net

[3] Nourang, A., Sadeghian, S., Eghbali, M., & Hajian, A. (2010). Supply chain analysis with system dynamics approach case study: medical equipment manufacturer. *7th international conference on industrial engineering, Tehran. Iran.* Civilica. (**In Persian**). https://civilica.com/doc/93563

[4] Taleizadeh, A. A., Niaki, S. T. A., & Wee, H.-M. (2013). Joint single vendor--single buyer supply chain problem with stochastic demand and fuzzy lead-time. *Knowledge-based systems*, *48*, 1–9. https://doi.org/10.1016/j.knosys.2013.03.011

[5] Pahlevani Ghomi, M., & Amiri, M. (2016). Presentation of a bi-level model for pricing and order planning in tri-echelon supply chain. *Modern research in decision making*, *1*(1), 27–53. (**In Persian**). https://journal.saim.ir/article_20703_77b1932b3eec9f094a2b103001f0dff1.pdf?lang=en

[6] Jafarnejad, A., Safari, h., Azar, A., & Ebrahimi, S. A. (2015). Optimal management of supply chain orders using activity-based costing, robust optimization and dynamic simulation. *Journal of operations research and its applications*, *12*(2), 19-30. (**In Persian**). https://jamlu.liau.ac.ir/browse.php?a_id=1145&sid=1&slc_lang=fa

[7] Heidari, J., & Zoghabi Ahmadi, P. (2016). Co-ordination of ordering policy and service level in a decentralized supply chain with uncertainty in delivery times. *Supply chain management*, *18*(52), 60-66. (**In Persian**). https://scmj.ihu.ac.ir/article_203595_6ad66950294d6b139ce5a8b8da00835d.pdf

[8] Ghasemi, P., Abolghasemian, M., & others. (2023). A Stackelberg game for closed-loop supply chains under uncertainty with genetic algorithm and gray wolf optimization. *Supply chain analytics*, *4*, 100040. https://doi.org/10.1016/j.sca.2023.100040

[9] Jahangiri, S., Abolghasemian, M., Pourghader Chobar, A., Nadaffard, A., & Mottaghi, V. (2021). Ranking of key resources in the humanitarian supply chain in the emergency department of iranian hospital: a real case study in COVID-19 conditions. *Journal of applied research on industrial engineering*, *8*(Special Issue), 1–10. https://doi.org/10.22105/jarie.2021.275255.1263

[10] Samieifard, M., Abolghasemian, M., & Pourghader Chobar, A. (2024). The impact of innovation, performance, and e-commerce development in the online shop on online marketing: A case study in the industry. *Interdisciplinary journal of management studies*, *18*(1), 1–17. https://doi.org/10.22059/ijms.2024.358619.675818

[11] Rezaei Kallaj, M., Abolghasemian, M., Moradi Pirbalouti, S., Sabk Ara, M., & Pourghader Chobar, A. (2021). Vehicle routing problem in relief supply under a crisis condition considering blood types. *Mathematical problems in engineering*, *2021*(1), 7217182. https://doi.org/10.1155/2021/7217182

[12] Chaharsooghi, S. K., & Heydari, J. (2010). Supply chain coordination for the joint determination of order quantity and reorder point using credit option. *European journal of operational research*, *204*(1), 86–95. https://doi.org/10.1016/j.ejor.2009.10.007

[13] Chaharsooghi, S. K., Heydari, J., & Kamalabadi, I. N. (2011). Simultaneous coordination of order quantity and reorder point in a two-stage supply chain. *Computers & operations research*, *38*(12), 1667–1677. https://doi.org/10.1016/j.cor.2011.02.012

[14] Arkan, A., & Hejazi, S. R. (2012). Coordinating orders in a two echelon supply chain with controllable lead time and ordering cost using the credit period. *Computers & industrial engineering*, *62*(1), 56–69. https://doi.org/10.1016/j.cie.2011.08.016

[15] Rezaei, A., Aghsami, A., & Rabbani, M. (2021). Supplier selection and order allocation model with disruption and environmental risks in centralized supply chain. *International journal of system assurance engineering and management*, *12*(6), 1036–1072. https://doi.org/10.1007/s13198-021-01164-1

[16] Liotta, G., Kaihara, T., & Stecca, G. (2014). Optimization and simulation of collaborative networks for sustainable production and transportation. *IEEE Transactions on Industrial Informatics*, *12*(1), 417-424. https://doi.org/10.1109/TII.2014.2369351

[17] Pérez-Perales, D., Boza, A., Alarcón, F., & Gómez-Gasquet, P. (2024). Mathematical programming-based methodology for the evaluation of supply chain collaborative planning scenarios. *Annals of operations research*, *337*(1), 261–312. https://doi.org/10.1007/s10479-024-05917-6